Bir Sınıf Sturm-Liouville Operatörü için Düz ve Ters Problem Üzerine

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ÖZET

Çalışmada $[0,\infty)$ yarıekseninde

$$-y'' + q(x)y = \lambda^2 \rho(x) y \tag{1}$$

denklemi

$$y'(0) - hy(0) = 0 \tag{2}$$

sınır koşulu ile birlikte ele alınır, burada $q(x)$

$$\int_0^\infty (1+x)|q(x)|\,dx < \infty \tag{3}$$

koşulunu sağlayan gerçel fonksiyondur, h keyfi bir gerçel sayıdır, λ karmaşık bir parametredir, $\rho(x)$ sınırlı sayıda süreksizlik noktası olan pozitif bir parça sabitidir. Bu çalışmadada, (1) - (3) sınır değer problemi için saçılma teorisinin düz ve ters problemini araştırıyoruz . $\rho(x)=1$ durumunda saçılmanın ters probleminin tam çözümü [1-3] 'te verilmiştir. [4] ve [5] 'de, dönüşüm operatörü kullanılarak saçılmanın ters probleminin çözümü, $[0,a]$ and $[a,\infty)$ gibi aralıklarda iki ters problemin çözümüne indirilmiştir.

$\rho(x)\neq 1$ süreksiz durumda, (1) denkleminin Jost çözümünün yeni (üçgen olmayan) integral gösterimini kullanarak saçılmanın ters problemi [6,7]'de tamamen çözüldü. Bu durumda, $\rho(x)$ fonksiyonun süreksizliği, Jost çözümünün yapısını ve ters problemin temel denklemini güçlü bir şekilde etkiler. $q(x)=0$ olduğu durumda ters problemin çözümünün tekliği [7] ve [8]' de incelenmiştir. Parçalı-sabit katsayılı bir dalga denklemi için ters problem [9] ve [10]'da çözülmüştür.

Anahtar Kelimeler: Sturm-Liouville, Ters Problem, Özdeğerler.

A DIRECT AND INVERSE PROBLEM FOR A CLASS STURM-LIOUVILLE OPERATOR

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ABSTRACT

We consider the equation

$$-y'' + q(x)y = \lambda^2 \rho(x) y \tag{1}$$

on the half line $[0,\infty)$ with the boundary condition

$$y'(0) - hy(0) = 0 \tag{2}$$

where $q(x)$ is a real-valued function satisfying the condition

$$\int_0^\infty (1+x)|q(x)|\,dx < \infty. \tag{3}$$

h is an arbitrary real number, λ is a complex parameter, $\rho(x)$ is a positive piecewise-constant with a finite number of points of discontinuity. In this paper, we investigate the direct and inverse scattering problem for the boundary value problem (1)-(3). We note that in the case $\rho(x)=1$, the inverse problem of the scattering was completely solved in [1-3]. When $\rho(x)\neq 1$, in [4] and [5] solution of inverse scattering problem by using the transformation operator was reduced to solution of two inverse problems on the intervals $[0,a]$ and $[a,\infty)$.

The discontinuous version by using the new (nontriangular) integral representation of Jost solution of equation (1) completely solved by [6,7]. In this case the discontinuity of the function $\rho(x)$ strongly influences the structure of representation of the Jost solution and the main equation of the inverse problem. Uniqueness of the solution of the inverse problem for (1) when $q(x)=0$ was given by [7] and [8]. Inverse problem for a wave equation with a piecewise-constant coefficient was solved by [9] and [10].

Key Words: Sturm-Liouville, Inverse Problem, Eigenvalues.

KAYNAKLAR - REFERENCES

[1] Marchenko V.A., Sturm-Liouville operators and their application, Naukova Dmka, Kiev, 1977; English trans. Birkhauser, 1986.

[2] Levitan B.M., On the solution of the inverse problem of quantum scattering theory, Mathematical Notes 1975,17(4):611-624.

[3] Levitan B.M., Inverse Sturm-Liouville problems. VSP, Zeist, The Netherlands; 1987.

[4] Gasymov M.G., The direct and inverse problem of spectral analysis for a class of equations with a discontinuous coefficient. In Non-Classical Methods in Geophysics. Edited by: Lavrentev M.M. Nauka, Novosibirsk, Russia; 1977:37-44.

[5] Darwish A.A., The inverse problem for a singular boundary value problem. New Zeland Journal of Mathematics 1993, 22: 37-66.

[6] Guseinov I.M., Pashaev RT: On an inverse problem for a second-order differential equation. Uspekhi Mat. Nauk, 2002, Volume 57, Issue 3(345), 147–148

[7] Mamedov Kh.R., Uniqueness of Solution of the Inverse Problem of Scattering Theory for Sturm – Liouville Operator with Discontinuous Coefficient, 2006, Proceedings of IMM of NAS of Azerbaijan **2006**, 24, 163-172.

[8] Tikhonov A.N., On the uniqueness of the solution of the problem of electric prospecting. Doklady Akademii Nauk SSSR 1949, 69: 797-800.

[9] Alimov S.A.,Tihonov's works on inverse problems for the Sturm-Liouville equation, Uspekhi Mat. Nauk, 1976, 31(6): 84-88. English translation in: Russian Mathematical Surveys, vol. 31, pp. 87–92, 1976.

[10] Lavrentev M.M., An inverse problem for the wave equation with a piecewise-constant coefficient. Sibirskii Matematicheskii Zhurnal 1992, 33(3):101-111, 219. translation in: Siberian Mathematical Journal, vol. 33, no. 3, pp. 452–461, 1992.

[11] Sedipkov A., Inverse spectral problem for the Sturm-Liouville operator with discontinuous potential,Journal of Inverse and Ill-Posed Problems 20(2), 2012.