On Some Applications of Measures of Noncompactness

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ÖZET

Seminerimizde, non-kompaktlık ölçülerinin teorisi ve uygulamalarına kısa bir genel bakış yapacağız. Klasik non-kompaktlık ölçüleri tartışılacak ve özellikleri kıyaslanacaktır. Genel metrik uzaylarda ve lineer uzaylarda non-kompaktlık ölçülerinin inşası, yoğunluk operatörü için sabit noktanın varlığı sonuçları ile, tanımlanacaktır. Ayrıca, klasik sonuçların birçok genelleştirmesinden bahsedildiği gibi, lineer denklemler, diferensiyel denklemler, integral denklemler ve denklemlerin ortak çözümleri gibi analizin değişik uygulamaları tartışılacaktır. Banach uzayları arasında kompakt operatörlerin karakterizasyonundaki en etkili yöntem non-kompaktlığın Hausdorff ölçüsünün uygulanmasıdır. Bu seminerimizde keyfi BK-uzayını c_0, c, ℓ_∞ ve ℓ_1 dizi uzaylarına dönüştüren sonsuz matrisler ile ifade edilebilen operatörlerin non-kompaktlığın Hausdorff ölçüsü ve operator normları için üst sınırlar ve bazı özdeşlikler verilecektir. Birçok lineer operatör, fonksiyon uzaylarındaki integral operatörler ve matris operatörleri olarak ifade edilebilir [1]. Son araştırmalar ile non-kompaktlık, integral denklemlerin [2] ve diferensiyel denklemlerin [3] sonsuz sistemlerinin çözümünde uygulamalar bulmuştur.

Key Words: Dizi uzayları; non-kompaktlık ölçüsü; kompakt operatörler; sonsuz diferensiyel denklem sistemleri.

ABSTRACT

In this talk, we present a brief survey of theory and applications of measures of noncompactness. The classical measures of noncompactness are discussed and their properties are compared. The approaches for constructing measure of noncompactness in a general metric or linear space are described, along with the classical results for existence of fixed point for condensing operators. Also several generalization of classical results are mentioned and their applications in various problems of analysis such as linear equation, differential equations, integral equations and common solutions of equations are discussed. The most effective way in the characterization of compact operators between the Banach spaces is applying the Hausdorff measure of noncompactness. In this talk, we present some identities or estimates for the operator norms and the Hausdorff measures of noncompactness of certain operators given by infinite matrices that map an arbitrary BK-space into the sequence spaces c_0, c, ℓ_∞ and ℓ_1 Many linear compact operators may be represented as matrix operators in sequence spaces or integral operators in function spaces [1]. Recently the measures of noncompactness are applied in solving infinite system of integral equations [2] and differential equations [3] in sequence spaces.

Key Words: Sequence spaces; measures of noncompactness; compact operators; infinite system of differential equations.

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